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Q.1. Define different types of relations and Equivalence class.

Soln. I Empty relation: → A relation R on a set A is called empty relation, if no element of A is related to any element of A. $R = \phi \subset A \times A$ is the empty relation.
For example, Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a+b = 8\}$. Then, no element of A is related to any element of A. So, R is an empty relation.

II Universal relation: → A relation R on a set A is called universal relation, if each element of A is related to every element of A. Thus, $R = A \times A$ is the universal relation.
For example, Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : a+b \leq 6\}$, then every element of A is related to every element of A. So, R is a universal relation on A. i.e. $R = A \times A$.

III A relation R on a set A is called
(a) reflexive, if $(a, a) \in R$ for every $a \in A$.
(b) Symmetric, if $(a, b) \in R \Rightarrow (b, a) \in R$
(c) Transitive, if $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$.

IV Equivalence ~~class~~ relation: → A relation R on a set A is called an equivalence relation, if it is reflexive, symmetric and transitive.

V Equivalence Class: → Let R be an equivalence relation on a non-empty set A and let $a \in A$. Then, $[a]$ is the equivalence class determined by a and defined as

$$[a] = \{x \in A : (a, x) \in R\}.$$

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Exercise 1.1

Q.1 Determine whether each of the following relations are reflexive, symmetric and transitive.

(I) Relation R in the set $A = \{1,2,3,\cdots 13,14\}$ defined as $R = \{(x,y) : 3x - y = 0\}$.

Soln: It is given that, $A = \{1,2,3,4,\cdots 12,13,14\}$ and R is $= \{(x,y) : 3x-y=0\}$ for reflexive relation $(x,x) \in R \ \forall x \in A$

If $y = x$, $3x - y = 0 \Rightarrow 3x - x = 0 \Rightarrow 2x \neq 0$. So, R is not reflexive. Then $(x,x) \notin R$.

~~If $(x,y) \in R \Rightarrow 3x-y=0$, then $3y-x \neq 0$~~

(b) It is given that $R = \{(x,y) : 3x-y=0\}$ for symmetric relation $(x,y) \in R \Rightarrow (y,x) \in R$.

If $(x,y) \in R \Rightarrow 3x - y = 0$, then $3y - x \neq 0 \Rightarrow (y,x) \notin R$, so, R is not symmetric.

For example, if $x=1$, $y=3$. $3\times1-3=0$, $3\times3-1=9-1=8\neq0$

(c) For transitive, if $(x,y) \in R \Rightarrow 3x-y=0$, $(y,z) \in R \Rightarrow 3y-z=0$, then $3x-z\neq0$. So, R is not transitive.

~~For Example~~ For example, when $x=1$, $y=3$, $z=9$, then $3\times1-3=0$, $3\times3-9=0$, $3\times1-9\neq0$.

(II) Relation R in set N of natural numbers defined as $R = \{(x,y) : y = x+5 \text{ and } x<4\}$.

Soln: Here, we see that, $A = N$, the set of natural numbers and $R = \{(x,y) : y = x+5, x<4\} = \{(x, x+5) : x \in N \text{ and } x<4\}$
$= \{(1,6), (2,7), (3,8)\}$

(a) For reflexive $(x,x) \in R \ \forall x$, putting $y = x$, $x \neq y+5 \Rightarrow (1,1) \notin R$. So, R is not reflexive.

(b) For symmetric $(x,y) \in R \Rightarrow (y,x) \in R$, Putting $y = x+5$, then $x \neq y+5 \Rightarrow (1,6) \in R$, but $(6,1) \notin R$. So, R is not symmetric.

(c) For transitivity; $(x,y) \in R, (y,z) \in R \Rightarrow (x,z) \in R$, if $y = x+5$, $z = y+5$, then, $z \neq x+5$. Since, $(1,6) \in R$ and there is no order pair in R which has 6 as the first element same in the case for $(2,7)$ and $(3,8)$. So, R is not transitive.

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